DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

340. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Let $S_{n-1}=1^{n-1}+2^{n-1}+3^{n-1}+\ldots+(n-1)^{n-1}$. Find n if $S_{n-1}-(n-1)$ is a multiple of n^2.

No complete solution of this problem has been received.

341. Proposed by O. L. CALLICOTT, Gettysburg, South Dakota.

Prove that the sum of the series, $\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+\ldots$ to infinity $=$ the sum of the series $\frac{1}{2}.1+\frac{1}{2^2}.\frac{1}{2}+\frac{1}{2^3}.\frac{1}{3}+\frac{1}{2^4}.\frac{1}{4}+\ldots$ to infinity.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

$$\frac{1}{(n+1)\ (n+2)}\equiv\frac{1}{n+1}-\frac{1}{n+2}.$$

Hence, by substituting for n, 0, 1, 2, 3, ..., and adding, we have

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots.=\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots.=\log 2.$$

Now $-\log(1-x)=x+\frac{x^2}{2}+\frac{x^3}{3}+\ldots$, which for $x=\frac{1}{2}$, becomes

$$-\log(\tfrac{1}{2})=\log 2=\tfrac{1}{2}+\tfrac{1}{2}(\tfrac{1}{2})^2+\tfrac{1}{3}(\tfrac{1}{2})^3+\ldots$$

This proves the proposition.

Also solved by V. M. Spunar.

GEOMETRY.

367. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

The tangents from a point A to a circle are bisected by a line XYZ, which cuts a chord in X and the tangents at its extremities in Y, Z. Show that $XAY=XAZ$, or $XAY=\pi-XAZ$. Also, reciprocate with respect to A.

Solution by S. LEFSEHETZ, Fellow in Clark University.

Let B, D be the points where the chord and circle meet. From Z draw ZC parallel to YD. The $\angle CBZ=\angle CDY=\angle BCZ$.

$\therefore CZ=BZ$. Also $ZB=ZA$, and $YB=YA$, hence is radical axis of circle and point A.

$\therefore \frac{XY}{XZ}=\frac{YD}{ZC}=\frac{YD}{ZB}=\frac{YA}{ZA}$, which shows that AX is one of the bisectors of the angle YAZ. The other is AX', X' being the point where the polar of X relative to the circle cuts XYZ.

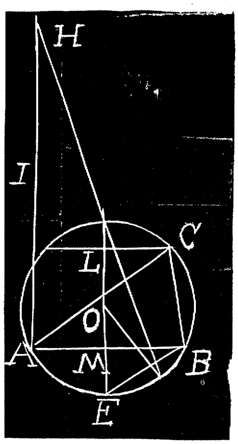
Reciprocally, given a line XZ which does not cut a circle, O, the points of this line which are conjugate relatively to O form on YZ a division in involution. There is a point A from which segments like XX are seen under a right angle, and YZ is the radical axis of this point and the circle.

368. Proposed by G. I. HOPKINS, Professor of Mathematics and Astronomy, Manchester High School, Manchester, N. H.

It is required to construct the triangle having given, base, vertical angle, and ratio of its altitude to sum of the other two sides.

Solution by the late G. B. M. ZERR, Ph.

Let $AB=a$ be a given base; C, the given vertical angle; x, y, z the two other sides and altitude, respectively; m the given ratio so that $x+y=mz$.



Now $xy\sin C=az$, $a^2=x^2+y^2-2xy\cos C$, $x^2+2xy+y^2=m^2z^2$. Hence $a^2=m^2z^2-2az\cot\frac{1}{2}C$.

$$\therefore z=\frac{a}{m}(\cot\tfrac{1}{2}C+\operatorname{cosec}\tfrac{1}{2}C)=\frac{a}{m}\cot\tfrac{1}{4}C.$$

Construction: On AB construct a segment containing the angle C. Draw the diameter DE perpendicular to AB. Bisect arc EB with radius OF at point F. Draw AI perpendicular to AB, and draw FD to meet AI produced in H. Then $\angle AHF=\angle EDF=\frac{1}{4}C$, and $AH=a\cot\frac{1}{4}C$. Find z a fourth proportional to M, AH and unity.

Take $ML=z$, and through L draw LC perpendicular to ML, meeting the circle in C. Join CA, CB, giving the required triangle ABC.

Also solved by A. H. Holmes.